

# Making confining strings out of mesons

Ryuichiro Kitano, Mitsutoshi Nakamura, and Naoto Yokoi

*Department of Physics, Tohoku University, Sendai 980-8578, Japan*

## Abstract

The light mesons such as  $\pi$ ,  $\rho$ ,  $\omega$ ,  $f_0$ , and  $a_0$  are possible candidates of magnetic degrees of freedom, if a magnetic dual picture of QCD exists. We construct a linear sigma model to describe spontaneous breaking of the magnetic gauge group, in which there is a stable vortex configuration of vector and scalar mesons. We numerically examine whether such a string can be interpreted as the confining string. By using meson masses and couplings as inputs, we calculate the tension of the string as well as the strength of the Coulomb force between static quarks. They are found to be consistent with those inferred from the quarkonium spectrum and the Regge trajectories of hadrons. By using the same Lagrangian, the critical temperature of the QCD phase transition is estimated, and a non-trivial flavor dependence is predicted. We also discuss a possible connection between the Seiberg duality and the magnetic model we studied.

# 1 Introduction

The dual Meissner effect is a plausible explanation of the color confinement in QCD [1, 2]. The condensation of the magnetic monopole,  $\langle m \rangle \neq 0$ , makes the QCD vacuum to be in the dual superconducting phase, where color fluxes sourced by quarks are squeezed into tubes, explaining the linear potential between quarks.

In the QCD vacuum, there is another interesting phenomenon called the chiral symmetry breaking. It is believed that the quark-antiquark pair condenses in the vacuum, and the  $SU(N_f)_L \times SU(N_f)_R$  symmetry is spontaneously broken down to a diagonal  $SU(N_f)_V$  group.

Since two condensations,  $\langle m \rangle \neq 0$  and  $\langle \bar{q}q \rangle \neq 0$ , happen in the same dynamics, these two may be related. Indeed, lattice simulations of finite temperature QCD suggest that deconfinement and chiral symmetry restoration happen at similar temperatures [3, 4]. This leads us to consider a simple unified picture:

$$\langle \bar{q}q \rangle \neq 0 \leftrightarrow \langle \bar{m}m \rangle \neq 0,$$

where “ $\leftrightarrow$ ” represents a non-abelian electric-magnetic duality. If the magnetic “monopoles”  $m$  and  $\bar{m}$  carry flavor quantum numbers, the condensations  $\langle m \rangle = \langle \bar{m} \rangle \neq 0$  describe Higgsing of the magnetic gauge group as well as chiral symmetry breaking. This phenomenon has been observed in supersymmetric gauge theories [5–8].

In this hypothesis, the “monopole” condensations give masses to magnetic gauge bosons and simultaneously provide massless pions as the Nambu-Goldstone bosons. There is in fact such a structure in the real hadron world. It has been known that the masses and interactions of the pions and the vector mesons such as the  $\rho$  and the  $\omega$  mesons are well described by a spontaneously broken  $U(N_f)$  gauge theory [9]. (See [10] for an earlier discussion on the description of the vector mesons as gauge fields.) The vector mesons and the pions are respectively interpreted as the gauge fields and the uneaten Nambu-Goldstone bosons. Therefore, we are naturally lead to consider a possibility that the  $\rho$  and  $\omega$  mesons are actually the magnetic gauge bosons of QCD. Along this line, it has been demonstrated recently that QCD regularized into an  $\mathcal{N} = 1$  supersymmetric theory has such a magnetic description via the Seiberg duality [11], where the magnetic Higgs fields  $m$  and  $\bar{m}$  are the dual scalar quarks [12].

The Higgs model of the  $U(N_f)$  gauge theory contains vector and scalar fields as well as strings as solitonic objects [13–16]. Since the string carries a magnetic flux in the magnetic picture, it can naturally be identified as the confining string via the electric-magnetic duality.

We examine in this paper whether such an identification works at the quantitative level. By using hadron masses and coupling constants as inputs, one can calculate the string tension and the Coulomb force between static quarks. We obtain values which are consistent with those inferred from the quarkonium spectrum and the Regge trajectories in the hadron spectrum.

We write down a linear sigma model which includes the vector mesons, the pions and the scalar mesons in the next section. A vortex configuration in the model is constructed in Section 3, and we compare the energy of the monopole-antimonopole system to the experimentally measured potential between a quark and an antiquark in Section 4. The critical temperature of the QCD transition is estimated in Section 5. The identification of the vector mesons as magnetic gauge bosons is motivated by recent discussions in supersymmetric gauge theories [12, 17, 18]. (See also [19] for an earlier discussion.) We extend the discussion and propose a new interpretation in Section 6.

## 2 Magnetic linear sigma model

The magnetic picture of a confining gauge theory is supposed to be a Higgs model of some gauge theory. We apply this principle in QCD, and construct a model to describe Higgsing of the magnetic gauge group as well as chiral symmetry breaking.

### 2.1 Lagrangian

We propose the following Lagrangian to describe the magnetic picture of QCD. It is a  $U(N_f)$  gauge theory, and the Lagrangian possesses the  $U(N_f)_L \times U(N_f)_R$  chiral symmetry. The vacuum expectation values (VEVs) of the Higgs fields,  $H_L$  and  $H_R$ , break the chiral symmetry down to the diagonal subgroup,  $U(N_f)_V$ , providing massless Nambu-Goldstone bosons identified as pions and  $\eta$ . The  $\eta$  meson (or  $\eta'$  in the three-flavor language) can obtain a mass through a term which breaks axial  $U(1)$  symmetry explicitly such as  $\det(H_L H_R)$  although we ignore it in this paper. The VEVs of the Higgs fields give masses to  $U(N_f)$  gauge bosons. We identify these massive gauge bosons as the  $\rho$  and the  $\omega$  mesons\*. The Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}^{(\omega)}F^{(\omega)\mu\nu} - \frac{1}{4}F_{\mu\nu}^{(\rho)a}F^{(\rho)\mu\nu a} \\ & + \frac{f_\pi^2}{2}\text{Tr} [|D_\mu H_L|^2 + |D_\mu H_R|^2] \\ & - V(H_L, H_R). \end{aligned} \tag{1}$$

---

\*In the three-flavor language, one should include  $K^*(892)$  and  $\phi(1020)$  in the vector mesons.

The first and the second terms represent the kinetic terms of the  $U(1)$  and the  $SU(N_f)$  parts of the  $U(N_f)$  gauge bosons:  $\omega_\mu$  and  $\rho_\mu^a$ , respectively. The Higgs fields  $H_L$  and  $H_R$  are  $N_f \times N_f$  matrices which transform as

$$H_L \rightarrow g_L H_L g_H^{-1}, \quad H_R \rightarrow g_H H_R g_R^{-1}, \quad (2)$$

under the  $U(N_f)_L$ , the gauged  $U(N_f)$ , and the  $U(N_f)_R$  group elements,  $g_L$ ,  $g_H$ , and  $g_R$ , respectively. The covariant derivatives are, therefore, given by

$$D_\mu H_L = \partial_\mu H_L + i g_2 H_L \rho_\mu^a T^a + i g_1 Q \omega_\mu H_L, \quad (3)$$

$$D_\mu H_R = \partial_\mu H_R - i g_2 \rho_\mu^a T^a H_R - i g_1 Q \omega_\mu H_R. \quad (4)$$

Here, we normalized the  $SU(N_f)$  generators in the fundamental representation,  $T^a$ , and the  $U(1)$  charge,  $Q$ , such that

$$\text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab}, \quad (5)$$

and

$$Q = \sqrt{\frac{1}{2N_f}}. \quad (6)$$

The most general potential terms consistent with the symmetries are given by

$$\begin{aligned} V(H_L, H_R) = & f_\pi^4 \left[ \frac{\lambda_0 - \lambda_A}{8N_f} \left( \text{Tr}(H_L H_L^\dagger) + \text{Tr}(H_R^\dagger H_R) - 2N_f \right)^2 \right. \\ & + \frac{\lambda_A}{8} \left\{ \text{Tr} \left[ (H_L^\dagger H_L + H_R H_R^\dagger)^2 \right] - 4 \left( \text{Tr}(H_L H_L^\dagger) + \text{Tr}(H_R^\dagger H_R) \right) \right\} \\ & + \frac{\lambda' - \lambda''}{8N_f} \left( \text{Tr}(H_L H_L^\dagger) - \text{Tr}(H_R^\dagger H_R) \right)^2 \\ & \left. + \frac{\lambda''}{8} \text{Tr} \left[ (H_L^\dagger H_L - H_R H_R^\dagger)^2 \right] \right], \quad (7) \end{aligned}$$

where we assumed the parity invariance under  $H_L \leftrightarrow H_R$ . This potential stabilizes  $H_L$  and  $H_R$  at

$$\langle H_L \rangle = \langle H_R \rangle = \mathbf{1}. \quad (8)$$

At the vacuum,  $4N_f^2$  degrees of freedom in  $H_L$  and  $H_R$  break up to  $N_f^2$  massless Nambu-Goldstone bosons,  $N_f^2$  longitudinal modes of the gauge bosons,  $N_f^2$  massive scalar particles, and  $N_f^2$  massive pseudoscalar particles. The decay constant of the Nambu-Goldstone particles

is given by  $f_\pi$  at tree level. The gauge group is completely broken and the unbroken global symmetry is vectorial  $U(N_f)_V$ .

The physical modes at the vacuum can be classified by the representations of  $U(N_f)_V$ , the spin and the parity. The masses of the physical modes are given by

$$\text{singlet Nambu-Goldstone boson } (\eta): \quad m_\eta = 0, \quad (9)$$

$$\text{adjoint Nambu-Goldstone boson } (\pi): \quad m_\pi = 0, \quad (10)$$

$$\text{singlet vector } (\omega): \quad m_\omega^2 = g_1^2 f_\pi^2, \quad (11)$$

$$\text{adjoint vector } (\rho): \quad m_\rho^2 = g_2^2 f_\pi^2, \quad (12)$$

$$\text{singlet scalar } (f_0): \quad m_S^2 = 2\lambda_0 f_\pi^2, \quad (13)$$

$$\text{adjoint scalar } (a_0): \quad m_A^2 = 2\lambda_A f_\pi^2, \quad (14)$$

$$\text{singlet pseudoscalar}: \quad m_{PS}^2 = 2\lambda' f_\pi^2, \quad (15)$$

$$\text{adjoint pseudoscalar}: \quad m_{PA}^2 = 2\lambda'' f_\pi^2, \quad (16)$$

at tree level. Terms with  $\lambda'$  and  $\lambda''$  are not very important in the following discussion<sup>†</sup>.

Hereafter, we take

$$g_1 = g_2 \equiv g, \quad (17)$$

as the  $\rho$  and  $\omega$  mesons have similar masses.

---

<sup>†</sup> The pseudoscalar particles are, in fact,  $CP$  even, and thus they are exotic states which are absent in the hadron spectrum. One should take large  $\lambda'$  and  $\lambda''$  to make the exotic states heavy so that the model can be a low-energy effective theory of QCD. We thank M. Harada, V.A. Miransky, and K. Yamawaki for discussion on this point.

## 2.2 Vector mesons and pions

When we integrate out the massive scalar and pseudoscalar mesons, the model reduces to a non-linear sigma model of Ref. [9]. The matching at tree level gives  $a = 1$ , where  $a$  is a parameter in the low-energy Lagrangian:

$$\mathcal{L} \ni \frac{(1-a)f_\pi^2}{4} \text{Tr} [|\partial_\mu(U_L U_R)|^2]. \quad (18)$$

The unitary matrices  $U_L$  and  $U_R$  are fields to describe the Nambu-Goldstone modes including the ones eaten by the gauge bosons. The transformation properties of  $U_L$  and  $U_R$  under the gauge and flavor groups are the same as  $H_L$  and  $H_R$ , respectively. From the low energy data, the preferred value of  $a$  is estimated to be  $a \sim 2$  with an error of 15% [20]. Although there is a factor of two difference from the prediction, this discrepancy can be explained by including quantum corrections and/or higher dimensional operators. As discussed in Ref. [20], the quantum correction makes the Lagrangian parameter  $a(\Lambda)$  approaches to unity when we take  $\Lambda$  to be large, such as  $a(\Lambda) \simeq 1.33 \pm 0.28$  for  $\Lambda = 4\pi f_\pi \sim 1$  GeV. Moreover, the quantum corrections from the scalar loops give positive contributions to the gauge boson masses, that further reduces the  $a(\Lambda)$  parameter. Therefore, one can think of the Lagrangian in Eq. (1) as the one defined at a high energy scale such as the mass scale of the scalar mesons.

However, the large quantum corrections result in predictions which depend on the choice of input physical quantities when we work at tree level, although the differences should be canceled after including quantum corrections. In this case, one should choose a set of physical quantities which gives small enough coupling constants so that the use of the perturbative expansion is valid and the tree-level results are reliable.

The Lagrangian has four parameters relevant for the discussion:  $g$ ,  $f_\pi$ ,  $\lambda_0$ , and  $\lambda_A$ . The  $\lambda_0$  and  $\lambda_A$  parameters can be obtained from the scalar masses as we discuss later. The gauge coupling constant  $g$  and the  $f_\pi$  parameter can be estimated from two of physical quantities. The well-measured physical quantities which can be used as input parameters are [20]:

$$g_\rho = (340 \text{ MeV})^2, \quad g_{\rho\pi\pi} = 6.0, \quad F_\pi = 92 \text{ MeV}, \quad m_\rho = 770 \text{ MeV}, \quad (19)$$

where  $g_\rho$  and  $g_{\rho\pi\pi}$  are the decay constant and the coupling to two pions of the  $\rho$  meson measured by  $\rho \rightarrow e^+e^-$  and  $\rho \rightarrow \pi\pi$  decays, respectively, and  $F_\pi$  is the decay constant of the pion. The relations to the Lagrangian parameters at tree level are given by

$$g_\rho = g f_\pi^2, \quad g_{\rho\pi\pi} = \frac{g}{2}, \quad F_\pi = f_\pi, \quad m_\rho = g f_\pi. \quad (20)$$

Among them, the pair to give the smallest gauge coupling is  $g_\rho$  and  $m_\rho$  such as

$$g = \frac{m_\rho^2}{g_\rho} = 5.0, \quad f_\pi = \frac{g_\rho}{m_\rho} = 150 \text{ MeV}. \quad (21)$$

The value  $g = 5.0$  means that the loop expansion parameter,  $g^2 N_f / (4\pi)^2$ , is of order 30% whereas other choices of input quantities give 90 – 210% for  $N_f = 2$ . Therefore, the choice above is unique to make a quantitative prediction. Indeed, the values in Eq. (21) are close to the ones evaluated at one-loop level. In Ref. [20], the parameters at a scale  $\Lambda \sim 1 \text{ GeV}$  is obtained to be  $g(\Lambda) \sim 3.3 - 4.2$ ,  $f_\pi(\Lambda) \sim 130 - 150 \text{ MeV}$ , and  $a(\Lambda) \sim 1.0 - 1.5$ , which reproduce all the physical quantities in Eq. (19). We use the values of  $g$  and  $f_\pi$  in Eq. (21) in the following discussion. However, we should bear in mind that there are theoretical uncertainties at the level of a factor of two in the results obtained at the classical level.

### 2.3 Scalar mesons

In the hadron spectrum, there are light scalar mesons, such as  $\sigma$ ,  $\kappa$ ,  $f_0(980)$  and  $a_0(980)$ , which have not been understood as  $q\bar{q}$  states in the quark model since they are anomalously light. We propose to identify them as the Higgs bosons in this linear sigma model. We do not consider heavier scalar mesons as candidates since otherwise the formulas in Eqs. (13) and (14) indicate that the coupling constants are large and the perturbation theory would not be applicable.

By taking the masses of  $f_0(980)$  and  $a_0(980)$  as input quantities<sup>‡</sup>, *i.e.*,

$$m_S = m_A = 980 \text{ MeV}, \quad (22)$$

Eqs. (13) and (14) give the coupling constants  $\lambda_0$  and  $\lambda_A$  as

$$\sqrt{\lambda_0} = \sqrt{\lambda_A} = 4.6, \quad (23)$$

at tree level, where  $f_\pi$  in Eq. (21) is used. We use these values of coupling constants for later calculations.

## 3 Vortex strings

Since the model has a spontaneously broken gauged  $U(1)$  factor, there is a vortex string as a classical field configuration. The string carries a quantized magnetic flux. Below we construct a solution with a unit flux, which will be identified as the confining string.

---

<sup>‡</sup>Since  $\sigma$  and  $\kappa$  are quite broad resonances, we do not use their masses as inputs.

There have been similar approaches to the confinement in QCD. The Ginzburg-Landau models (the magnetic Higgs models) are constructed from phenomenological approaches [14, 21, 22] or based on the QCD Lagrangian [23, 24] through the abelian projection [25], and the stable vortex configurations are identified as the confining string. In supersymmetric theories, there have been numbers of discussion on the vortex configurations [15, 16, 26–31]. In particular, the non-abelian string [15, 16], which we discuss shortly, has been extensively studied as a candidate of the confining string.

Our model combines Higgsing of the magnetic gauge group and chiral symmetry breaking. As discussed in the previous Section, the model parameters are fixed by physical quantities such as masses and couplings of hadrons. Therefore, the properties of the strings such as the string tension can be evaluated quantitatively. Below, we explicitly construct a classical field configuration of the vortex string.

### 3.1 Non-abelian vortex solutions

In this model, there are string configurations called the non-abelian vortices which carry the minimal magnetic flux. By defining the following gauge field,

$$A_{\mu}^{ij} = \sqrt{2} (Q\omega_{\mu}\delta_{ij} + T_{ij}^a\rho_{\mu}^a), \quad (24)$$

there is a vortex configuration made of, *e.g.*, the  $i = j = 1$  component rather than the overall  $U(1)$  gauge field  $\omega_{\mu}$ . Compared to the string solution made of  $\omega_{\mu}$ , this non-abelian string carries only  $1/N_f$  of the magnetic flux and thus it is stable.

In constructing the vortex configurations, we follow the formalism and numerical methods of Ref. [32], where the potential between a monopole and an anti-monopole is evaluated numerically in the abelian-Higgs model. Classical field configurations are constructed by numerically solving field equations while imposing the gauge field to behave as the Dirac monopoles [33] as approaching to their locations.

We consider a non-abelian vortex solution, where the magnetic flux is sourced by a Dirac-monopole and a Dirac-antimonopole configurations of the  $A_{\mu}^{ij}$  gauge field with  $i = j = 1$ , representing non-abelian monopole configurations. These monopole and anti-monopole are not present as physical states in the model of Eq. (1), and we introduce them as field configurations with an infinite energy, *i.e.*, static quarks<sup>§</sup>. The object we construct

---

<sup>§</sup>In  $U(N)$  gauge theories with Higgs fields in the adjoint representation, there are monopoles as solitonic objects which are identified as junctions of vortices [34, 35] rather than the endpoints. The monopoles we are considering should not be confused with such configurations.



here, therefore, corresponds to a bound state of heavy quarks such as the charmonium and the bottomonium. In order to describe light mesons, the light quarks should be present somewhere in the whole framework. We discuss a possible framework in Section 6.

In the cylindrical coordinate,  $(\rho, \varphi, z)$ , where the monopole and the antimonopole located on the  $z$ -axis at  $z = \pm R/2$ , we denote  $(A_D)_\mu^{ij}$  as the configuration to describe the monopole-antimonopole system. They are given by

$$(A_D)_0^{ij} = 0, \quad (25)$$

$$A_D^{ij} = 0, \quad \text{except for } i = j = 1, \quad (26)$$

and

$$A_D^{11} = a_D \hat{\varphi} = -\frac{N_{\text{flux}}}{\sqrt{2}g} \frac{1}{\rho} \left[ \frac{z - R/2}{[\rho^2 + (z - R/2)^2]^{1/2}} - \frac{z + R/2}{[\rho^2 + (z + R/2)^2]^{1/2}} \right] \hat{\varphi}. \quad (27)$$

The number of the flux,  $N_{\text{flux}}$ , is quantized as  $N_{\text{flux}} \in \mathbb{Z}$  by the Dirac quantization condition [33]. Equivalently, the magnetic charge of the monopole is quantized as

$$q_m = \frac{4\pi N_{\text{flux}}}{\sqrt{2}g}. \quad (28)$$

The gauge field is well-defined everywhere except for the interval  $-R/2 \leq z \leq R/2$  on the  $z$ -axis. The Dirac quantization condition ensures that the interval is covered in a different gauge. For constructing a vortex configuration, the following ansatz are taken:

$$A_\mu^{ij} = A_\mu^i \delta^{ij}, \quad A_\mu^i = (A_D)_\mu^{ii} + a_\mu^i, \quad (29)$$

$$a_0^i = 0, \quad \mathbf{a}^i = a^i(\rho, z) \hat{\varphi}, \quad (30)$$

$$(H_L)_{ij} = (H_R)_{ij} = \phi_i(\rho, z) \delta_{ij}, \quad \phi_i = \phi_i^*. \quad (31)$$

With the ansatz, the Lagrangian is reduced to

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} \sum_i F^{i\mu\nu} F_{\mu\nu}^i \\ & + f_\pi^2 \sum_i (\partial_\mu \phi_i)^2 + \frac{f_\pi^2}{2} g^2 \sum_i \phi_i^2 (A_\mu^i)^2 \\ & - \frac{\lambda_0}{2N_f} f_\pi^4 \left( \sum_i \phi_i^2 - N_f \right)^2 \\ & - \frac{\lambda_A}{2N_f} f_\pi^4 \left( N_f \sum_i \phi_i^4 - \left( \sum_i \phi_i^2 \right)^2 \right), \end{aligned} \quad (32)$$

and the field equations are obtained as

$$\nabla^2 \phi_i - \frac{g^2}{2} (a^i + a_D \delta^{i1})^2 \phi_i = \frac{\lambda_0}{2N_f} \left( \sum_j \phi_j^2 - N_f \right) \phi_i + \frac{\lambda_A}{2N_f} \left( N_f \phi_i^2 - \sum_j \phi_j^2 \right) \phi_i, \quad (33)$$

$$\left( \nabla^2 - \frac{1}{\rho^2} \right) a^i = \frac{g^2}{2} (a^i + a_D \delta^{i1}) \phi_i^2, \quad (34)$$

where we take the unit of

$$\sqrt{2} f_\pi = 1. \quad (35)$$

For  $i \neq 1$ ,  $a^i = 0$  is the solution.

The potential energy between the monopole and the anti-monopole is given by

$$\begin{aligned} V(R) = & -\frac{2\pi N_{\text{flux}}^2}{g^2 R} \\ & + \int d^3x \left[ -\frac{g^2}{4} \phi_1^2 (a^1 + a_D) a^1 - \frac{\lambda_0}{8N_f} \left( \left( \sum_i \phi_i^2 \right)^2 - N_f^2 \right) \right. \\ & \left. - \frac{\lambda_A}{8N_f} \left( N_f \sum_i \phi_i^4 - \left( \sum_i \phi_i^2 \right)^2 \right) \right]. \end{aligned} \quad (36)$$

The first term comes from the magnetic Coulomb potential,  $V_{\text{Coulomb}} = -q_{\text{mag}}^2/4\pi R$ . The second term is the contribution from the non-trivial field configurations, and gives the linear potential between a monopole and an antimonopole for a large  $R$ . The self-energies of the Dirac monopoles are subtracted, and thus this expression provides a finite quantity.

For  $\lambda_0 = \lambda_A$ , which is the case as in Eq. (23), the problem simplifies to the case of the abelian string. The field equations gives

$$\phi_i = 1, \quad \text{for } i \neq 1, \quad (37)$$

as solutions and the equations for  $\phi_1$  and  $a^1$  becomes

$$\nabla^2 \phi_1 - \frac{g^2}{2} (a^1 + a_D)^2 \phi_1 = \frac{\lambda_0}{2} (\phi_1^2 - 1) \phi_1, \quad (38)$$

$$\left( \nabla^2 - \frac{1}{\rho^2} \right) a^1 = \frac{g^2}{2} (a^1 + a_D) \phi_1^2. \quad (39)$$

The potential energy is in this case given by

$$V(R) = -\frac{2\pi N_{\text{flux}}^2}{g^2 R} + \int d^3x \left[ -\frac{g^2}{4} \phi_1^2 (a^1 + a_D) a^1 - \frac{\lambda_0}{8} (\phi_1^4 - 1) \right]. \quad (40)$$

The  $N_f$  dependence disappears from the potential energy.

### 3.2 Numerical results

We numerically solve Eqs. (38) and (39) by following the procedure explained in Ref. [32]. The partial differential equations are solved by using the Gauss-Seidel method. The obtained field configurations are used to evaluate the potential energy in Eq. (40).

In the unit of Eq. (35), the potential energy  $V(R)$  times the gauge boson mass  $m_\rho$  can be obtained as a function of  $m_\rho R$ . In this normalization, we have a single parameter  $\kappa$  defined by

$$\kappa = \frac{m_S}{\sqrt{2}m_\rho} = \frac{\sqrt{\lambda_0}}{g} = \frac{\sqrt{\lambda_A}}{g}. \quad (41)$$

This corresponds to the Ginzburg-Landau parameter of superconductors. The numerical results are shown in Fig. 1, where the potential energies for  $N_{\text{flux}} = 1$  are drawn with four choices of parameters,  $\kappa = 0.1, 0.9, 1.7$ , and  $2.5$ . We see a linear potential in a large  $R$  region. By fitting the slope of the linear regime, one can extract the string tension  $\hat{\sigma}$  in the unit of Eq. (35). We show in Fig. 2 the tension  $\hat{\sigma}$  as a function of  $\kappa$ . These results are all consistent with Ref. [32], except that the unit of the flux is different due to the non-abelian feature of the vortex. For  $\kappa = 1/\sqrt{2}$ , the field equations reduce to a set of first order differential equations whose solutions are known as the BPS state. In this case, the tension is simply given by  $\hat{\sigma} = \pi$ , which we have confirmed with an accuracy of  $0.1 - 0.2$  percent.

## 4 Comparison to QCD data

Now we compare the numerical results with data from experimental measurements. We identify the non-abelian Dirac monopoles with the minimal magnetic charge,  $N_{\text{flux}} = 1$ , as static quarks, since otherwise the string with  $N_{\text{flux}} = 1$  is stable and such a stable string is absent in QCD. The potential between a quark and an antiquark with a distance  $R$  can be parametrized by the following form:

$$V(R) = -\frac{A}{R} + \sigma R. \quad (42)$$

This potential, called the Cornell potential, well fits the quarkonium spectrum with parameters:

$$A \sim 0.25 - 0.5, \quad \sqrt{\sigma} \sim 430 \text{ MeV}. \quad (43)$$

A similar value of the string tension  $\sigma$  is obtained from the Regge trajectories of the hadron spectrum. The lattice simulations also reproduce the shape of the potential with  $A \sim 0.25 -$

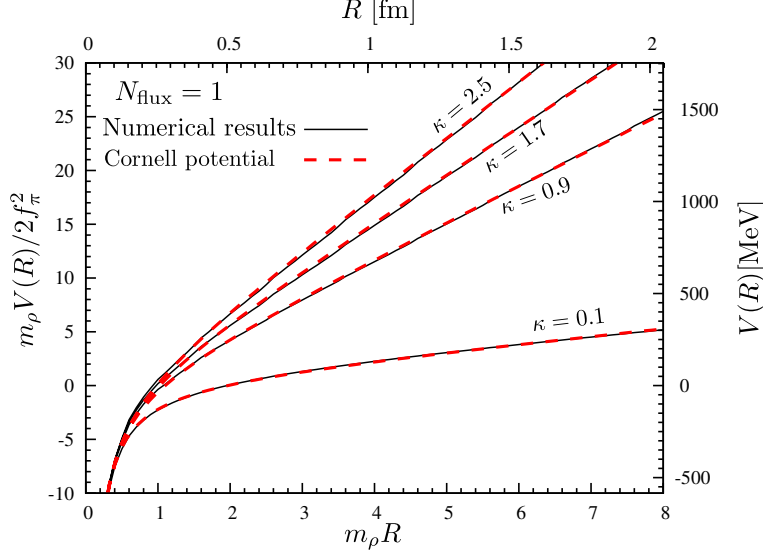


Figure 1: Potential energy of the monopole-antimonopole system for  $\kappa = 0.1, 0.9, 1.7$  and  $2.5$ . The fittings with the Cornell potential are superimposed (dashed lines).

$0.4$  [36–38] and  $\sqrt{\sigma}/m_\rho \sim 0.50 - 0.55$  [39]. In perturbative QCD, at tree level, the Coulomb part  $V \sim -A/R$  is obtained from the one-gluon exchange between quarks. At a higher loop level, the shape of the potential approaches to the form in Eq. (42) [40]. Computations at three-loop level have been performed recently in Refs. [41, 42], and it is reported that the result is in good agreement with lattice simulations up to a distance scale  $R \lesssim 0.25$  fm [41]. See, for example, Ref. [43] for a review of the static QCD potential.

The Cornell potential also well fits the numerically obtained potential in the previous section. We superimpose the fittings with the Cornell potential in Fig. 1 as dashed lines.

#### 4.1 Coulomb potential

In the electric picture, *i.e.*, in QCD, the Coulomb part  $V \sim -A/R$  is obtained with

$$A = \frac{N_c^2 - 1}{2N_c} \frac{g_s^2}{4\pi}, \quad (44)$$

where the strong gauge coupling  $g_s$  depends on  $R$  through renormalization.

By duality, in the magnetic picture, the Coulomb term is accounted by a magnetic Coulomb force between monopoles. By using the magnetic charge in Eq. (28) with  $N_{\text{flux}} = 1$ , the coefficient is given by

$$A = \frac{q_m^2}{4\pi} = \frac{2\pi}{g^2}. \quad (45)$$

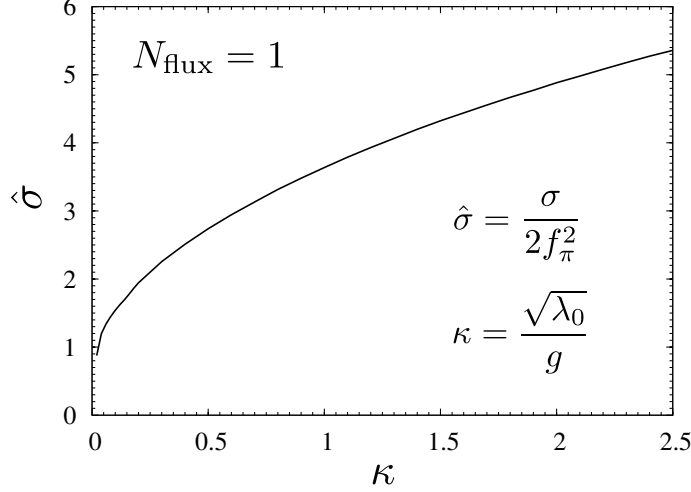


Figure 2: The string tension  $\hat{\sigma}$  in the unit of  $2f_\pi^2$  as a function of the Ginzburg-Landau parameter  $\kappa$ .

This corresponds to the first term in Eq. (40). Using the value of  $g$  in Eq. (21), we obtain

$$A = 0.25. \quad (46)$$

The value is consistent with Eq. (43). This is already an interesting non-trivial test of the hypothesis that the vector mesons are the magnetic gauge fields.

Note here that the Coulomb term in Eq. (40) arises from a solution of the classical field equations with boundary conditions given by the Dirac monopoles. Although the vacuum is in a Higgs phase, the Coulomb force dominates when the distance  $R$  is small compared to the inverse of the gauge boson mass. In the world-sheet theory of the string, it has been known that the Coulomb force can be reproduced as the Lüscher term which stems from the boundary conditions of the string world sheet [44]. Interestingly, the Lüscher term gives  $A = \pi/12 \sim 0.26$  which is pretty close to the above estimation.

## 4.2 Linear potential

As we have seen already, the linear potential is obtained as in Fig. 1. The normalized string tension  $\hat{\sigma}$  is shown in Fig. 2 as a function of  $\kappa$ . From Eqs. (21), (23) and (41), the  $\kappa$  parameter is given by

$$\kappa = 0.90. \quad (47)$$

With this value, we obtain from Fig. 2,

$$\hat{\sigma} = 3.5. \quad (48)$$

By using  $f_\pi$  in Eq. (21) to recover the mass dimension, we obtain

$$\sqrt{\sigma} = 400 \text{ MeV}. \quad (49)$$

This is close to  $\sqrt{\sigma}$  in Eq. (43). The prediction is not very sensitive to  $\kappa$ . For example,  $\kappa = 0.6 - 1.2$  gives  $\sqrt{\sigma} = 360 - 420 \text{ MeV}$ . Although we expect a large theoretical uncertainty from quantum corrections, it is interesting to note that the estimated string tension is in the right ballpark. The hypothesis that the  $\rho$  and  $\omega$  mesons as magnetic gauge bosons and light scalar mesons as the Higgs bosons is found to be consistent with the experimental data.

It is important to notice that there is no dependence on  $N_f$  in the field equations (38), (39) or in the expression of the QCD potential (40). It is essential to have this property that the string is non-abelian. The dimensionless quantity  $\sqrt{\sigma}/m_\rho$  is, in this case, predicted to be  $N_f$  independent, which is consistent with the results from the lattice QCD [39].

## 5 QCD phase transition

At a finite temperature, QCD phase transition takes place. The lattice simulations support that deconfinement and chiral symmetry restoration happen at similar temperatures. The chiral transition temperature has been computed in lattice simulations, and found to be  $T_c \sim 150 - 160 \text{ MeV}$  [3, 4] for physical quark masses.

A simple estimate of the transition temperature is possible in the magnetic model in Eq. (1). The deconfinement and the restoration of the chiral symmetry both correspond to the phase transition to the vacuum with  $H_L = H_R = 0$ , which is stabilized by thermal masses at a finite temperature. When we define the transition temperature  $T_c$  to be the one at which the Higgs fields become non-tachyonic at the origin, the temperature is obtained to be [45]

$$T_c = \sqrt{\frac{8}{\eta N_f}} f_\pi, \quad (50)$$

where the factor  $\eta$  is a dimensionless quantity given by

$$\eta = 1 + \frac{2m_\rho^2}{m_S^2} + \frac{2m_{PS}^2 + m_S^2}{3m_S^2}, \quad (51)$$

at the lowest level of perturbation. Each term in the  $\eta$  parameter represents the contribution to the thermal masses of the Higgs fields from different particles. The first term, the unity,

is the contribution from the scalar mesons. One should add up all the particles which obtain masses from the VEVs of  $H_L$  and  $H_R$ . The estimation of  $\eta$  is quite non-trivial since there are particles which we did not consider, such as nucleons, and also the summation should be weighted by the abundance in the thermal bath, which may be affected by their large thermal masses, *i.e.*, there may be large higher order corrections.

By putting  $f_\pi$  in Eq. (21), we obtain

$$T_c = \begin{cases} 170 \text{ MeV} \times \left(\frac{\eta}{3}\right)^{-1/2}, & (N_f = 2), \\ 140 \text{ MeV} \times \left(\frac{\eta}{3}\right)^{-1/2}, & (N_f = 3). \end{cases} \quad (52)$$

The value  $\eta \sim 3$  seems to give temperatures consistent with ones from lattice simulations. It is interesting that  $\eta \sim 3$  is obtained from Eq. (51) when we take  $m_{PS}$  around the cut-off scale,  $\Lambda \sim 1 \text{ GeV}$ .

The formula in Eq. (50) predicts that the transition temperature is inversely proportional to  $\sqrt{N_f}$ . This is numerically consistent with the flavor dependence of  $T_c$  studied in Ref. [39] for two and three flavors in the chiral limit. There,  $T_c$  is obtained to be  $173 \pm 8 \text{ MeV}$  and  $154 \pm 8 \text{ MeV}$  for two and three flavors, respectively. A simulation with a larger number of  $N_f$  should be able to test this prediction.

## 6 Non-supersymmetric duality from the Seiberg duality

The assumption in the whole framework is the electric-magnetic duality between the  $SU(N_c)$  gauge theory with  $N_f$  massless quarks and  $U(N_f)$  gauge theory with bosonic Higgs fields. The replacement of  $N_c$  in the gauge group with  $N_f$  is familiar in supersymmetric gauge theories. For example, the Seiberg duality in the  $\mathcal{N} = 1$  supersymmetric theories replaces  $SU(N_c)$  gauge group by  $SU(N_f - N_c)$  in the magnetic picture. We explain here a possible connection between the Lagrangian in Eq. (1) and the Seiberg duality, which is discussed in Ref. [12]. We extend the discussion of Ref. [12] regarding the vortex string and interpretations of constituent quarks.

It is obvious that the non-supersymmetric QCD can be obtained from supersymmetric QCD's by adding masses to superpartners and send them to infinity. What is non-trivial is if a vacuum in the theory with small masses of superpartners is continuously connected to the non-supersymmetric theory when we send the masses to large values. Such a continuous path may or may not exist depending on the space of parameters defined by a supersymmetric theory to start with. Recently, it is found in Ref. [12] that there is an explicit model which

	$SU(N_c)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$SU(N_c)_V$	$U(1)_{B'}$	$U(1)_R$
$Q$	$N_c$	$N_f$	1	1	1	0	$(N_f - N_c)/N_f$
$\overline{Q}$	$\overline{N_c}$	1	$\overline{N_f}$	-1	1	0	$(N_f - N_c)/N_f$
$Q'$	$N_c$	1	1	0	$\overline{N_c}$	1	1
$\overline{Q}'$	$\overline{N_c}$	1	1	0	$N_c$	-1	1

Table 1: Quantum numbers in the electric picture.

reduces to QCD in a limit of parameters and has a vacuum with the same structure as the low energy QCD in a region of parameters where the Seiberg duality can be used. By hoping that the region extends to the QCD limit, one can study non-perturbative features of QCD, such as strings, at the classical level in the dual picture.

The proposed mother theory is  $\mathcal{N} = 1$  supersymmetric QCD with  $N_c$  colors and  $N_f + N_c$  flavors. By giving supersymmetric masses to the extra  $N_c$  flavors and soft supersymmetry breaking masses for gauginos and scalar quarks, one obtains non-supersymmetric QCD with  $N_c$  colors and  $N_f$  flavors. The global symmetries and quantum numbers are listed in Table 1, where  $SU(N_c)$  is the gauge group. The  $U(1)_{B'}$  symmetry is absent in the actual QCD, and will be spontaneously broken in the vacuum we discuss later. In order to avoid the appearance of the unwanted Nambu-Goldstone mode associated with this breaking, we gauge  $U(1)_{B'}$ . The  $SU(N_c)_V$  group is also an artificially enhanced symmetry, and thus we gauge it. Since the added gauge fields only interact with extra flavors, the limit of large mass parameters still gives the non-supersymmetric QCD we wanted.

The magnetic picture of the mother theory is an  $SU(N_f)$  gauge theory with  $N_f + N_c$  flavors and meson fields. The particle content and the quantum numbers are listed in Table 2. It was found in Ref. [12] that there can be a stable vacuum outside the moduli space by the help of the soft supersymmetry breaking terms. The vacuum is at  $\langle q \rangle = \langle \bar{q} \rangle \neq 0$ , where  $SU(N_f) \times SU(N_f)_L \times SU(N_f)_R$  is spontaneously broken down to a single vectorial  $SU(N_f)_V$  symmetry, that is the isospin symmetry. The symmetry breaking provides massless pions and simultaneously gives masses to the  $SU(N_f) \times U(1)_{B'}$  gauge fields. Those massive gauge fields can be identified as the vector mesons,  $\rho$  and  $\omega$ .

Although the deformation with massive  $N_c$  flavors provides us with a QCD-like vacuum, there are several unsatisfactory features as noted in Ref. [12]. Here we discuss those issues and consider a possible interpretation. In the above discussion, it sounds somewhat strange that the  $U(1)_{B'}$  gauge field is identified as the  $\omega$  meson which is in the same nonet as the



	$SU(N_f)$	$SU(N_f)_L$	$SU(N_f)_R$	$U(1)_B$	$SU(N_c)_V$	$U(1)_{B'}$	$U(1)_R$
$q$	$N_f$	$\overline{N_f}$	1	0	1	$N_c/N_f$	$N_c/N_f$
$\overline{q}$	$\overline{N_f}$	1	$N_f$	0	1	$-N_c/N_f$	$N_c/N_f$
$\Phi$	1	$N_f$	$\overline{N_f}$	0	1	0	$2(N_f - N_c)/N_f$
$q'$	$N_f$	1	1	1	$N_c$	$-1 + N_c/N_f$	0
$\overline{q'}$	$\overline{N_f}$	1	1	-1	$\overline{N_c}$	$1 - N_c/N_f$	0
$Y$	1	1	1	0	$1 + \text{Adj.}$	0	2
$Z$	1	1	$\overline{N_f}$	-1	$\overline{N_c}$	1	$(2N_f - N_c)/N_f$
$\overline{Z}$	1	$N_f$	1	1	$N_c$	-1	$(2N_f - N_c)/N_f$

Table 2: Quantum numbers in the magnetic picture.

$\rho$  meson, whereas the  $U(1)_{B'}$  seems to have a completely different origin from the  $SU(N_f)$  magnetic gauge group. Second, in the particle content in Table 2, there are fields which have  $U(1)_B$  charges  $\pm 1$ , *i.e.*, “quarks.” These degrees of freedom do not match the picture of confinement since they look like free quarks. Finally, there is a vortex string associated with the spontaneous breaking of  $U(1)_{B'}$ , which we would like to identify as the QCD string. However, since the stability of the string is ensured by topology, it is stable even in the presence of the massless quarks. The real QCD string should be unstable since a pair creation of the quarks can break the string.

A possible interpretation is emerged from the consideration of the origin of  $U(1)_{B'}$  in the magnetic picture. As one can notice from the quantum numbers,  $U(1)_{B'}$  in the electric and magnetic pictures look different. In particular, the gauged global symmetry in the electric picture is  $U(N_c) \simeq (SU(N_c) \times U(1))/\mathbb{Z}_{N_c}$  whereas one cannot find a  $U(N_c)$  gauge group in the magnetic picture. This leads us to consider a possibility that there is an additional  $U(1)$  factor as a part of the magnetic gauge group. The actual magnetic gauge group is  $U(N_f)$ , and it is broken by a VEV of a field with the quantum number of  $Q'^{N_c} q^{N_f}$  so that  $U(1)_{B'}$  in the magnetic picture is an admixture of two  $U(1)$ ’s. Namely, the duality of the gauge group goes through an intermediate step:

$$\begin{aligned}
SU(N_c) \times U(N_c) \text{ (electric)} &\rightarrow U(N_f) \times U(N_c) \text{ (magnetic)} \\
&\rightarrow SU(N_f) \times SU(N_c)_V \times U(1)_{B'} \text{ (magnetic)}. \quad (53)
\end{aligned}$$

Under this assumption, when we send the gauge coupling of  $U(1) (\subset U(N_c))$  in the electric picture to be a large value, the gauge boson of the  $U(1)_{B'}$  factor in the magnetic picture is mostly the one from the  $U(N_f)$  magnetic gauge group. The identification of the  $\omega$  meson becomes reasonable since the origin is now the same as the  $\rho$  meson.

Since  $U(1)_{B'}$  is spontaneously broken by  $\langle q \rangle = \langle \bar{q} \rangle \neq 0$ , there is a stable vortex string which can be explicitly constructed as a classical field configuration in the magnetic picture. The duality steps (53) imply that there is another string in the magnetic picture: one associated with  $U(N_f)$  and another with  $U(N_c)$ . However, if we go back to the electric picture, there is only a single  $U(1)$  factor in  $U(N_c)$ , which can only give a single kind of string. This sounds like a mismatch of two descriptions.

We propose here that the  $U(N_f)$  string, made of  $q$ ,  $\bar{q}$ ,  $\rho$ , and  $\omega$ , is in fact unstable since the “quarks” can attach to the endpoints, and thus that is the one which should be identified as the QCD string. The  $U(N_c)$  string is stable, but should decouple in the QCD limit. As mentioned already, there are “quarks” in the magnetic picture,  $q'$ ,  $\bar{q}'$ ,  $Z$  and  $\bar{Z}$ . They are natural candidates of the “quarks” which attach to the  $U(N_f)$  string. In turn, if they are the degrees of freedom at the string endpoints, a linear potential prevents them to be in the one-particle states. Therefore, the “quarks” disappear from the spectrum. This interpretation seems to give resolutions to all the unsatisfactory features raised before: the nature of  $\omega$ , free quarks, and the stable string.

For this interpretation to be possible,  $q'$ ,  $\bar{q}'$ ,  $Z$  and  $\bar{Z}$  should carry magnetic charges of  $U(N_f)$  in addition to the quantum numbers listed in Table 2. Since we assume the electric-magnetic duality between the  $SU(N_c)$  and the  $U(N_f)$  gauge groups, it is equivalent to say that  $q'$ ,  $\bar{q}'$ ,  $Z$  and  $\bar{Z}$  should be colored under  $SU(N_c)$ , *i.e.*,  $Z$  and  $\bar{Z}$  are the quarks (the non-abelian monopoles in the magnetic picture) and  $q'$  and  $\bar{q}'$  are non-abelian dyons. It is interesting to notice that they indeed have  $N_c$  degrees of freedom.

In the  $SU(N_f) \times U(1)_{B'}$  magnetic gauge group, there is a  $U(1)$  factor which rotates a particular component of  $q_I$  and  $\bar{q}_I$ , where  $I$  is the index of the  $SU(N_f)$  gauge group. The vortex string associated with such a  $U(1)$  factor is called the non-abelian string and the one with the minimal magnetic flux is stable. Therefore, the “quarks” should attach to this string. When we take  $q_1$  is the one which rotates under the  $U(1)$  factor and normalize the charge of it as unity, the charges of other charged fields are listed in the left column of Table 3. By assuming that  $q$  and  $\bar{q}$  have no magnetic charges, the Dirac-Schwinger-Zwanziger condition [46,47] allows the magnetic charges listed in the right column of Table 3 as the minimal magnetic charges divided by  $(2\pi/e)$  with  $e$  being the gauge coupling constant. Interestingly, they agree with the “color charge” of  $SU(N_c)_V$  up to a normalization, which may be indicating that a part of  $SU(N_c)_V$  in the magnetic picture descends from the electric gauge group,  $SU(N_c)$ . For dynamical fields with both electric and magnetic quantum numbers, we loose the standard Lagrangian description of the model. However, since the

	(electric charges)/ $e$	(magnetic charges)/( $2\pi/e$ )
$q_1$	1	0
$\bar{q}_1$	-1	0
$q'_1$	$1 - 1/N_c$	1
$\bar{q}'_1$	$-1 + 1/N_c$	-1
$q'_{I \neq 1}$	$-1/N_c$	1
$\bar{q}'_{I \neq 1}$	$1/N_c$	-1
$Z$	$1/N_c$	-1
$\bar{Z}$	$-1/N_c$	1

Table 3: Electric and magnetic charges under a  $U(1)$  factor in  $SU(N_f) \times U(1)_{B'}$ .

sector of  $q$ ,  $\bar{q}$  (and  $\Phi$ ) is all singlet under  $SU(N_c)_V$  and is decoupled from the colored sector, there can be a Lagrangian to describe it, and we assume that is the model in Eq. (1).

It is amusing to see that many ingredients to describe the hadron world are present in this model, such as the vector mesons, the pions, the light scalar mesons, the QCD string, and the constituent quarks. This is somewhat surprising since the Seiberg duality is supposed to describe only massless degrees of freedom. The non-trivial success of the model may be indicating that the addition of  $N_c$  massive quarks is a right direction to fully connect the electric and magnetic pictures of  $\mathcal{N} = 1$  supersymmetric QCD.

## Acknowledgments

We would like to thank Yukinari Sumino for valuable information on the static QCD potential in perturbative QCD. RK would also like to thank Yutaka Ookouchi for useful discussion. RK is supported in part by the Grant-in-Aid for Scientific Research 23740165 of JSPS. NM is supported by the GCOE program “Weaving Science Web beyond Particle-Matter Hierarchy.”

## References

- [1] G. 't Hooft, talk at the E.P.S. Int. Conf. on High Energy Physics, Palermo, 23-28 June, 1975, PRINT-75-0836 (UTRECHT).
- [2] S. Mandelstam, Phys. Lett. **B53**, 476-478 (1975).
- [3] Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S. D. Katz, S. Krieg and K. K. Szabo, JHEP **0906**, 088 (2009) [arXiv:0903.4155 [hep-lat]].

- [4] A. Bazavov, T. Bhattacharya, M. Cheng, C. DeTar, H. T. Ding, S. Gottlieb, R. Gupta and P. Hegde *et al.*, arXiv:1111.1710 [hep-lat].
- [5] N. Seiberg, E. Witten, Nucl. Phys. **B431**, 484-550 (1994). [hep-th/9408099].
- [6] G. Carlino, K. Konishi, H. Murayama, JHEP **0002**, 004 (2000). [hep-th/0001036]; G. Carlino, K. Konishi, H. Murayama, Nucl. Phys. **B590**, 37-122 (2000). [hep-th/0005076]; G. Carlino, K. Konishi, S. Prem Kumar, H. Murayama, Nucl. Phys. **B608**, 51-102 (2001). [hep-th/0104064].
- [7] K. Konishi, G. Marmorini, N. Yokoi, Nucl. Phys. **B741**, 180-198 (2006). [hep-th/0511121].
- [8] A. Gorsky, M. Shifman, A. Yung, Phys. Rev. **D75**, 065032 (2007). [hep-th/0701040].
- [9] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985).
- [10] J. J. Sakurai, “Currents and Mesons,” (Univ. Chicago Press, Chicago, 1969).
- [11] N. Seiberg, Nucl. Phys. **B435**, 129-146 (1995). [hep-th/9411149].
- [12] R. Kitano, JHEP **1111**, 124 (2011) [arXiv:1109.6158 [hep-th]].
- [13] A. A. Abrikosov, Sov. Phys. JETP **5**, 1174 (1957) [Zh. Eksp. Teor. Fiz. **32**, 1442 (1957)].
- [14] H. B. Nielsen and P. Olesen, Nucl. Phys. B **61**, 45 (1973).
- [15] A. Hanany and D. Tong, JHEP **0307**, 037 (2003) [hep-th/0306150].
- [16] R. Auzzi, S. Bolognesi, J. Evslin, K. Konishi and A. Yung, Nucl. Phys. B **673**, 187 (2003) [hep-th/0307287].
- [17] Z. Komargodski, JHEP **1102**, 019 (2011) [arXiv:1010.4105 [hep-th]].
- [18] S. Abel and J. Barnard, arXiv:1202.2863 [hep-th].
- [19] M. Harada and K. Yamawaki, Phys. Rev. Lett. **83**, 3374 (1999) [hep-ph/9906445].
- [20] M. Harada and K. Yamawaki, Phys. Rept. **381**, 1 (2003) [hep-ph/0302103].
- [21] A. Jevicki and P. Senjanovic, Phys. Rev. D **11**, 860 (1975).
- [22] Y. Nambu, Phys. Rev. D **10**, 4262 (1974).
- [23] T. Suzuki, Prog. Theor. Phys. **80**, 929 (1988).

- [24] S. Maedan, Y. Matsubara and T. Suzuki, Prog. Theor. Phys. **84**, 130 (1990).
- [25] G. 't Hooft, Nucl. Phys. **B190**, 455 (1981).
- [26] M. J. Strassler, Prog. Theor. Phys. Suppl. **131**, 439 (1998) [arXiv:hep-lat/9803009].
- [27] M. Eto, L. Ferretti, K. Konishi, G. Marmorini, M. Nitta, K. Ohashi, W. Vinci, N. Yokoi, Nucl. Phys. **B780**, 161-187 (2007). [hep-th/0611313].
- [28] M. Shifman, A. Yung, Phys. Rev. **D76**, 045005 (2007). [arXiv:0705.3811 [hep-th]].
- [29] M. Eto, K. Hashimoto, S. Terashima, JHEP **0709**, 036 (2007). [arXiv:0706.2005 [hep-th]].
- [30] M. Shifman, A. Yung, Phys. Rev. **D83**, 105021 (2011). [arXiv:1103.3471 [hep-th]].
- [31] K. Hanaki, M. Ibe, Y. Ookouchi and C. S. Park, JHEP **1108**, 044 (2011) [arXiv:1106.0551 [hep-ph]].
- [32] J. S. Ball and A. Caticha, Phys. Rev. D **37**, 524 (1988).
- [33] P. A. M. Dirac, Phys. Rev. **74**, 817 (1948).
- [34] M. Shifman and A. Yung, Phys. Rev. D **70**, 045004 (2004) [hep-th/0403149].
- [35] A. Hanany and D. Tong, JHEP **0404**, 066 (2004) [hep-th/0403158].
- [36] T. T. Takahashi, H. Suganuma, Y. Nemoto and H. Matsufuru, Phys. Rev. D **65**, 114509 (2002) [hep-lat/0204011].
- [37] S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. D **68**, 054502 (2003) [hep-lat/0212039].
- [38] S. Aoki *et al.* [JLQCD Collaboration], Phys. Rev. D **78**, 014508 (2008) [arXiv:0803.3197 [hep-lat]].
- [39] F. Karsch, E. Laermann and A. Peikert, Nucl. Phys. B **605**, 579 (2001) [hep-lat/0012023].
- [40] Y. Sumino, Phys. Lett. B **571**, 173 (2003) [hep-ph/0303120]; Phys. Rev. D **76**, 114009 (2007) [hep-ph/0505034].
- [41] C. Anzai, Y. Kiyo and Y. Sumino, Phys. Rev. Lett. **104**, 112003 (2010) [arXiv:0911.4335 [hep-ph]].
- [42] A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Phys. Rev. Lett. **104**, 112002 (2010) [arXiv:0911.4742 [hep-ph]].

- [43] G. S. Bali, Phys. Rept. **343**, 1 (2001) [hep-ph/0001312].
- [44] M. Luscher, Nucl. Phys. B **180**, 317 (1981).
- [45] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974).
- [46] J. S. Schwinger, Phys. Rev. **144**, 1087 (1966); Phys. Rev. **173**, 1536 (1968).
- [47] D. Zwanziger, Phys. Rev. **176**, 1480 (1968).